Kindergarten Standards

1. Count by 1's, 5's and 10's.
2. Count on from specific numbers
3. Read and write numerals to twenty
4. Count objects up to 20.
5. Determine where there are more or less.
6. Compare objects using larger and smaller.
7. Follow directions utilizing numbers.
8. Show more than one way to display the same number.
9. Become familiar with numbers that combine to make 10.
10. Add and subtract numerals less than 10.
11. Count above 10 using the Ten-Frame.
12. Compare objects: longer, shorter, taller, wider
13. Measure weight, height, thickness, width.
14. Read basic graphs answering most, least and same questions.
15. Classify objects and explain rules of the classification.
16. Name Pattern Block Shapes (hexagon, rhombus, trapezoid, square, triangle)
17. Name geometric shapes within the classroom.
18. Follow above, below, beside, behind, next to and in front of directions.
19. Recognize various triangles, squares and rectangles.
20. Distinguish between flat and solid geometric shapes.
21. Know vertex, face and edge of solids.
22. Build spheres, cubes and cones.
23. Make rectangles, squares and triangles by paper folding.

Grade 1 Key Concepts

Number.

1. Match the number of objects with a written numeral.
2. Count to 120, starting at any number less than 120.
3. The two digits of a two-digit number represent the number of tens and the number of ones.
4. The three digits of a three-digit number represent the number of hundreds, number of tens and number of ones.
5. Count by 10's to 200.
6. Compare numbers using =, <, and >.
7. Mentally find ten more or ten less, without having to count.
8. Add one and two-digit numbers.
9. Subtract one and two-digit numbers.

Operations and Algebraic Thinking

10. Use addition and subtraction within 20 to solve word problems with two numbers.
11. Use addition and subtraction within 20 to solve word problems with three numbers.
12. Understand that problems such as 3 + 4 = 7 and 4 + 3 =7 are examples of the commutative property in addition.
13. Find the pairs of numbers that add to ten when adding strings of numbers. (3 + 7 = 10, 4 + 6 = 10, 8 + 2 = 10). (Associative property of addition)
14. Understand subtraction as a missing addend addition problem. 12-8= is the same as 8 + ? = 12.
15. Relate counting to addition and subtraction by using blank number line.
16. Use = and ≠ signs to write true and false addition and subtraction number sentences.
17. Solve addition and subtraction equations with missing numbers.

Measurement and Data

18. Order three objects from smallest to largest.
19. Read both analog and digital clock in hours and half-hours
20. Read and interpret graphs with up to three categories

Geometry

21. Identify attributes of basic geometric shapes.
22. Combine geometric shapes to make larger shapes.
23. Partition geometric shapes into equal shares describing the shares using the words halves, thirds, quarters, and fourths.

Grade 2 Key Concepts

Number

1. Understand place value with three-digit numbers.
2. Compare two three-digit numbers using <,>, and =.
3. Know that 100 is ten 10's and 1000 is ten 100's.
4. Read and write numbers to 1000 in standard and expanded form.
5. Count by 2's, 5's, 10's and 100's.
6. Fluently add and subtract numbers with one regrouping.
7. Add up to four two-digit numbers using place value strategies.
8. Show addition and subtraction, within 1000, using place value drawings.

Operations and Algebraic Thinking

9. Solve two-step addition and subtraction problems.
10. Be fluent with addition and subtraction problems up to 20.
11. Know odd and even numbers. Two equal numbers add to make an even number..
12. Find the total number of objects arranged in an array.
13. Mentally add or subtract 10 from any number between 100 and 900.
14. Describe an easier way to solve addition problems with regrouping.
15. Describe an easier way to solve subtraction problems with regrouping.

Measurement and Data

16. Measure objects using rulers, tapes, yard and meter sticks. Write two different accurate measures for the same measurement. (i.e. measure to nearest inch, nearest ½ inch, etc.)
17. Estimate lengths.
18. Compare two measures, subtract the difference.
19. Solve addition and subtraction word problems using lengths.
20. Show how to solve addition and subtraction problems on a number line.
21. Read analog and digital clocks to nearest 5 minutes, including AM and PM.
22. Know how many days in a week, months in a year, minutes in an hour, years in a century.
23. Solve money word problems using dollar and cents signs.
24. Read and interpret graphs with whole numbers.

Geometry

25. Count the number of angles and sides in polygons.
26. Count the number of vertices, edges and faces in polyhedra.
27. Identify names of Pattern Block shapes (hexagon, trapezoid, rhombus, square, triangle), pentagons, and cubes. Define quadrilateral.
28. Divide squares and rectangles into halves, thirds, fourths. Describe the whole as three thirds, four fourths, two halves.

Grade 3 Key Concepts

Number

1. Round whole numbers to nearest 10,100 or 1000.
2. Fluently add and subtract with two regroupings.
3. Multiply one-digit numbers.
4. Multiply one-digit numbers by multiples of 10 (e.g. 9 x 80 =, 5 x 60 =).
5. Identify a fractional part of a whole.
6. Represent fractions on a number line.
7. Understand equivalent fractions.
8. Express whole numbers as fractions (e.g. 6 = 6/1, 6 = 12/2)
9. Compare two fractions with =, < and >.
10. Divide two-digit numbers by one-digit numbers.

Operations and Algebraic Thinking

11. Write word problems for multiplication number sentences.
12. Use three models to represent division concepts: area, share and repeated subtraction. (Area: if a rectangle has an area of 12 and the width is 3, what is the length?; Sharing: if there are 12 marshmallows shared by 3 people, how many will each person receive?, Repeated subtraction: If a person has 12 wheels and wants to make tricycles, how many trikes can they make?)
13. Solve multiplication and division problems.
14. Find the missing number in multiplication and division number sentences.
15. Understand associative and distributive properties of multiplication and division. [Associative – 4 x 15= (4 x 10 + 4 x 5)]. [Distributive – 2 x 8 x 5 = (2 x 5 + 2 x 8)]
16. Understand division as an unknown factor problem. (32÷8= is the same as 8 x ? = 32)
17. Solve 2-step word problems with +, -, x, ÷.
18. Identify arithmetic patterns is the addition and multiplication tables.

Measurement and Data

19. Tell and write time to nearest minute and measure time intervals in minutes.
20. Solve time word problems.
21. Measure and estimate liquid volumes and masses of objects.
22. Interpret picture and bar graphs.
23. Use a ruler to measure to the nearest ¼ inch.
24. Measure area in square units (square cm, square m, square inch, square foot, square yard)
25. Solve measurement word problems.
26. Measure perimeter of polygons.

Geometry

27. Understand properties of quadrilaterals. Know the names and attributes of squares, rhombuses, parallelograms, rectangles and trapezoids.
28. Divide shapes into parts with equal areas.

Grade 4 Key Math Concepts

(Based upon Common Core State Standards by Peggy McLean and Lee Jenkins)

Number

1. Write base ten numerals in written and expanded forms. (e.g. 342 is three hundred forty-two; 342 = 300 + 40 + 2).
2. Compare two base ten numerals with =, < or >.
3. Round multi-digit numbers to the million's place.
4. Add and subtract whole numbers with three regroupings.
5. Multiply two digits by two digits.
6. Multiply up to four digits by one digit.
7. Find whole-number quotients and remainders with up to four-digit dividends and one digit divisors.
8. Multiplying the numerator and denominator of a fraction by the same numeral creates an equivalent fraction.
9. Compare two fractions with different numerators and denominators with =, < or >. Create common denominator for the comparison.
10. Use Pattern Blocks to add and subtract basic fractions.
11. Use Pattern Blocks to add and subtract mixed numerals.
12. Solve fraction word problems.
13. Multiply fractions by whole numbers.
14. Solve word problems with whole numbers and fractions.
15. Express fractions with a denominator of 10 as an equivalent fraction with a denominator of 100
16. Use decimal notation for fractions with denominators of 100.
17. Use =, < and > to compare decimals.

Operations and Algebraic Thinking

18. Solve word problems that require making sense of remainders.
19. Write the factor pairs for numbers less than 100.
20. Designate numbers less than 100 as prime or composite.
21. Complete function tables.

Measurement and Data

22. Understand relative sizes of measurement units such as minutes in an hour, grams in a kilogram, cups in a quart, feet in a yard.
23. Solve measurement word problems involving simple fractions and decimals.
24. Solve area and perimeter problems.
25. Solve fraction problems with a number line.

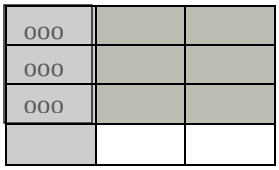
Geometry

26. Measure angles with a protractor.
27. Add and subtract angle measurements.
28. Identify right, acute and obtuse angles plus perpendicular and parallel lines.
29. Identify equilateral, isosceles and scalene triangles.
30. Recognize mirror symmetry of geometric shapes.

Grade 5 Key Math Concepts

Number

1. Complete a place value chart with both decimals and whole numbers.
2. Multiply and divide by 10.
3. Use exponents to denote powers of 10.
4. Read, write and compare decimals to the 1000^{th} place.
5. Write decimals in expanded form.
6. Round decimals to the nearest 1000^{th}.
7. Divide with two digit divisors.
8. Add, subtract, multiply and divide decimals.
9. Add and subtract fractions (including mixed numerals) without common denominators
10. Determine if fraction problem answers are reasonable.
11. Interpret a fraction as division of the numerator by the denominator.
12. Solve fraction multiplication problems by completing an array.



Multiplication is the rectangle or square formed by the intersection of two fractions. Here ¾ is overlapped with 1/3. The answer is 3/12, the portion where the ¾ and 1/3 overlap.

13. Understanding that multiplication by a number larger than 1 increases the size of the answer, whereas multiplying by a number smaller than 1 decreases the size of the answer.
14. Create equal multiplication problems by scaling factors. (e.g. 12 x 80 = 6 x 160)
15. Solve fraction multiplication word problems with mixed numerals.
16. Divide a whole number by a fraction and a fraction by a whole number. (4÷1/3 = 12; 1/3 ÷4 = 1/12.(Use egg cartons to understand).
17. Solve division of fraction word problems.

Operations and Algebraic Thinking

18. Use parentheses and brackets in numerical expressions.
19. Write simple verbal expressions as number sentences.
20. Find the prime factors of numbers between 2 and 50.
21. Complete function tables and graph on coordinate plane.

Measurement and Data

22. Convert different-sized standard measurements within a given measurement system (e.g. 1/15 hour = 4 minutes).
23. Interpret the line plot to find mean, median, mode and range.
24. Use 1 cm^3 cubes or 1 cubic inch cubes to measure volume of rectangular prisms.
25. Solve volume word problems.

Geometry

26. Plot points on a coordinate plane.
27. Solve word problems with a coordinate plane.
28. Classify polygons.
29. Classify polyhedra.

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Grade 6 Key Math Concepts

Number System

1. Fluently divide multi-digit numbers using the standard algorithm.
2. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.
3. Find the GCF of two whole numbers less than or equal to 100 and LCM of two whole numbers less than or equal to 12.
4. Identify Prime Numbers up to 100 and find the prime factorization of numbers up to 100.
5. Use the distributive property to express a sum of two whole numbers 1-100.
   Ex. 36 + 8 = 4(9 + 2) and 3(4 + 5) = 12 + 15 = 27
6. Understand a rational number(any # that can be expressed as a fraction of 2 integers where the denominator is not = 0) as a point on the number line.
7. Understand the absolute value of a rational number as its distance from 0 on the number line. Interpret absolute value in a real-world situation such as an account balance of -30 dollars = absolute value of 30 which is the size of the debit in dollars.
8. Compute quotients of fractions and solve real-world problems involving division of fractions.
9. Find integers on a number line and recognize opposites Ex. The opposite of the opposite of a number is the number such as –(-3) = 3 and zero is its own opposite.
10. Determine the distance between 2 integers on a number line
11. Work simple operations using integers by discovering the rules.
12. Solve real-world problems with integers.
13. Write, interpret, and explain statements of order for rational numbers in real-world contexts. Ex. -3 degrees Celsius > -7 degrees Celsius to express the fact that -3 degrees Celsius is warmer than -7.
14. Plot integers on the coordinate plane, understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane such as (-2,-3) is in quadrant 3.
15. Identify the new point if a point is rotated in the coordinated plane by 90, 180, 270 and 360 degrees.

Expressions and Equations

16. Write and evaluate numerical expressions involving whole-number exponents.
17. Write and read expressions involving variables. Ex. “Subtract y from 5” as 5 – y. Use words such as “twice”, “double”, “less”, “difference”, “increase”……
18. Identify the parts of an algebraic expression (sum, term, product, factor, quotient, coefficient, variable, constant).
19. Evaluate expressions given values for the variable. $4x + 5x – 2x$, when x = 5 or x^5, x = 2 Include expressions that arise from formulas such as $V = s^3$ or $A = 6s^2$
20. Simplify expressions using the distributive property and learn to combine like terms. Ex. 2(x + 4) = 2x + 8 and 3x + 5x + 2x = 10x and Y + Y + Y = 3Y
21. Create a table of values for algebraic expressions. Inputs vs Outputs using x and y in a table

format. This leads into the idea of an equation. Identify a correct table of values for an expression.

22. Solve one step equations. (addition, subtraction, and multiplication)
23. Understand solving an equation as a process of answering a question: which values from a specified set, if any, make the equation true? Pick out a table of values to fit an equation.
24. Understand how a table of values represents ordered pairs that can be plotted in the coordinate plane to represent the graph of an equation. Ex. $y = 2x$ or $y = x + 2$
25. Write inequalities of the form $x > 4$ or $4 < x$ that represent real-world situations and understand that there are infinitely many solutions.
26. Graph basic inequalities on a number line diagram.

Ratios and Proportional Relationships

27. Understand the concept of a ratio and use ratio language to describe the ratio relationship between two quantities.
28. Calculate basic unit rates. Ex. We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.
29. Use tables to compare ratios and find missing values. Ex. 2 laps = 10 minutes 4 laps = 20 minutes so 10 laps = ? minutes, make a table. Connect this table back to tables for equations and show how you could plot the values on a coordinate plane to represent the data.
30. Use cross multiplication and solving basic multiplication equations to solve unit rate problems. Ex. If it took 7 hours to mow 4 lawns then at that rate, how many lawns could be mowed in 35 hours?
31. Use ratio reasoning to convert measurement units. 4 quarts = 1 gal so 36 quarts = ?gal
32. Solve percent problems given a part and a percent. Ex. Suppose we know that 8 students earned an A in a class and we know that is 30%. How many total students were there in the class? Set up an equation and solve: 30% of ? = 8 $.30x = 8$ It becomes a multiplication equation and we divide to solve.

Geometry

33. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.
34. Find the volume of a right rectangular prism with fractional edge lengths $V = lwh$ and $V = bh$
35. Represent 3-D figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures
36. Draw polygons in the coordinate plane given coordinates for the vertices. Use the coordinates to find the length of sides. Determine area of polygons in the coordinate plane.

Statistics and Probability

37. Determine if statistics is needed to answer a question.
38. Determine the mean, median, mode, range, interquartile range, and mean absolute deviation.
39. Plot data sets on a number line, dot plots, box plots, and histograms. Analyze the data in these charts and answer questions.

Pre-Algebra (Grade 7) Key Math Concepts

Number System

1. Convert rational numbers to decimal form using long division. (know that a rational number terminates in 0's or eventually repeats)
2. Compare rational numbers (>,<,=).
3. Graph rational numbers on a number line.
4. Introduce irrational numbers for 8th grade
    a. Classify as rational or irrational
5. Adding/Subtracting/Multiplying/Dividing rational numbers (including decimals, integers & fractions) without a calculator
    a. Adding a negative is equivalent to subtracting
    b. Subtracting a negative is equivalent to adding
    c. Subtracting is the same thing as adding a negative
    d. Additive Inverses=0
    e. Absolute Value
6. Multiply/Divide Integers.
7. Properties of addition and multiplication (additive inverse, associative, commutative, identity, & distributive).

Ratios and Proportional Relationships

8. Writing a ratio (in simplest form).
9. Find the unit rate & solve problems with the unit rate (including fractions: 1/2 a mile in ¼ of an hour).
10. Testing if two ratios are proportional (using a table and by graphing on coordinate plane).
11. Solve proportions using algebra.
12. Identify Constant of Proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
13. Represent proportions using equations.
14. Setting up proportions.
15. Simple Interest, Discount, Tax, Tip, Markups, Markdowns, % of change(increase/decrease), Commissions, Fees, % Error.

Expressions and Equations

16. Simplify expressions by combining like terms.
17. Simplify expressions by using the distributive property (Expanding).
18. Factor (5y+10)...5(y+2).
19. Write an equation to solve a word problem.
20. Use algebra to solve word problems (linear equations)(4y+3=27).
21. Also solve equations using the distributive property
    a. Also include fractions and decimals in the equations (-7 + 1/4z=0).

22. Solve up to 2-step inequalities
23. Translate a problem into an inequality
24. Graphing

Geometry

25. Similar Figures (missing sides).
26. Scale Drawings of geometric figures.
27. Difference between congruent and similar.
28. Area & perimeter (2-D Figures) Including regular polygons(up to nonagon) Find the length or width of a figure given its area or perimeter.
29. Circumference of circle
30. Surface Area
31. Volume
32. Cross Sections of 3D Figures (right rectangular prisms and right rectangular pyramids).
33. Draw geometric shapes with given conditions...focus on constructing triangles and if the conditions form a triangle.
34. Angle Relationships (complementary, supplementary, vertical, adjacent).
35. Use equations to find an unknown angle in a figure.
36. Lines cut by a transversal (interior, exterior, alternate, corresponding).

Statistics and Probability

37. Simple (Theoretical), Compound, and Experimental Probability
38. Use lists, tree diagrams, tables
39. Samples/Population
40. Measures of Central Tendency
41. Line Plots, Box and Whisker, Tables
42. Mean Absolute Deviation
43. Making predictions using data
44. Comparing data sets

Algebra 1 (Grade 8) Key Math Concepts

Number System

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.
2. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 x 10^8 the population of the world as 7 × 10^8 and determine that the world population is more than 20 times larger.*
4. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so ($5^{1/3}$)^3 must equal 5.*
5. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Expressions and Equations

6. Interpret expressions that represent a quantity in terms of its context.
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P*(1+*r*)n *as the product of P and a factor not depending on P.*
7. Use the structure of an expression to identify ways to rewrite it.
8. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
9. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x = a*, *a = a*, or *a = b* results (where *a* and *b* are different numbers).
10. Solve linear equations in one variable, including equations with coefficients represented by letters.
11. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
12. Create linear equations in one variable and use them to solve problems.
13. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
14. Create equations in two or more variables to represent relationships between quantities.
15. Solve inequalities in one variable.

16. Create inequalities in one variable and use them to solve problems.
17. Represent constraints by inequalities.
18. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
19. Graph equations on coordinate axes with labels and scales.
20. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b* using point-slope.
21. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
22. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots and factoring.
23. Factor a quadratic expression (all types) to reveal the zeros of the function it defines
24. Solve quadratic equations by using the Quadratic Formula (real solutions only).

Expressions and Equations – Systems of Linear Equations

25. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
26. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
27. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$.
28. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
29. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
30. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
31. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
32. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions

33. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
34. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
35. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
36. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
37. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
38. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Algebra 2 Key Math Concepts

Number and Quantity

The Complex Number System

1. Know there is a complex number i such that $i^2 = –1$, and every complex number has the form a + bi with a and b real.
2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. Find the conjugate of a complex number; use conjugates to find quotients of complex numbers.
4. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities

5. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
6. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
7. Add, subtract, and multiply matrices of appropriate dimensions.
8. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. Use inverse to solve systems.
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Arithmetic with Polynomials and Rational Expressions

11. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
12. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
13. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
14. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

15. Know and apply the Binomial Theorem for the expansion of $(x + y)n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
16. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Reasoning with Equations and Inequalities

17. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
18. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
19. Solve quadratic equations in one variable.
20. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
21. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
22. Solve absolute value, exponential, and logarithmic functions.
23. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Functions

Interpreting Functions

24. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
25. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
26. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
27. Graph linear and quadratic functions(including completing the square) and show intercepts, maxima, and minima.
28. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

29. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
30. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
31. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
32. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
33. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
34. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

35. Write a function that describes a relationship between two quantities.
36. Determine an explicit expression, a recursive process, or steps for calculation from a context.
37. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
38. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
39. Write arithmetic and geometric sequences both recursively and use them to model situations, and translate between the two forms.
40. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for $n \geq 1$.
41. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
42. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x–1) for $x \neq 1$.
43. Verify by composition that one function is the inverse of another.
44. Read values of an inverse function from a graph or a table, given that the function has an inverse.
45. Produce an invertible function from a non-invertible function by restricting the domain.
46. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents

Linear, Quadratic, and Exponential Models

47. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

48. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
49. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
50. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
51. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
52. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
53. Interpret the parameters in a linear or exponential function in terms of a context.

Geometry Key Math Concepts

Congruence

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Geometric Measurement and Dimension

12. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
13. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★
14. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

15. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
16. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
17. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Similarity, Right Triangles, and Trigonometry

18. Verify experimentally the properties of dilations given by a center and a scale factor:
    a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
19. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
20. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
21. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
22. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
23. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
24. Explain and use the relationship between the sine and cosine of complementary angles.
25. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Circles

26. Prove that all circles are similar.
27. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
28. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
29. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations

30. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
31. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
32. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
33. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★